Towards multi-refinement for the *LeanMachines* Framework

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Supervised by Frédéric Peschanski and Romain Demangeon

Introduction

The Event B method

- Formal method for state-based reactive systems

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  - Typed functional programming language,
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  - A proof assistant
- Specfication, implementation, proofs of corrections, all in one go !

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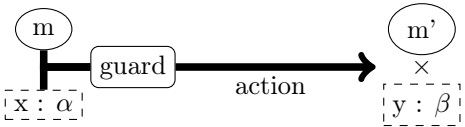
- Guarded transitions



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  - guard (m : M) (x : α) : Prop
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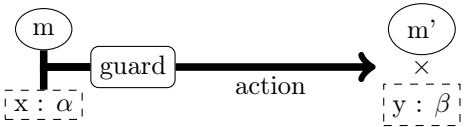

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- Guarded transitions
  - guard (m : M) (x : α) : Prop
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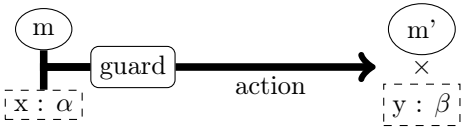
- Events **must** respect certain properties (**proof obligations** for the user)



Definition of a machine(formalisation)[1]

```
class Machine (CTX : Type u) (M) where
  context : CTX
  invariant : M → Prop
```

The Machine typeclass

[1]The code presented in these slides is a simplified version of the code of *LeanMachines*, complete version can be found here : https://github.com/lean-machines-central/lean-machines

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context : static part of the system (constants, properties about the constants)

invariant : properties the user wants to specify about the machine

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Definition of a machine (example)

Model of a bounded buffer

- The *size* of the buffer must be bounded by *maxSize*

```
structure BufContext where
  maxSize : Nat
  maxSizeProp : maxSize > 0

structure B0 (ctx : BufContext) where
  size : Nat
```

Definition of a machine (example)

Model of a bounded buffer

- The *size* of the buffer must be bounded by *maxSize*
- We **specify** this by instanciating the typeclass Machine

```
structure BufContext where
  maxSize : Nat
  maxSizeProp : maxSize > 0

structure B0 (ctx : BufContext) where
  size : Nat

instance: Machine BufContext (B0 ctx)
    where
  context := ctx
  invariant b0 := b0.size ≤ ctx.maxSize
```

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Adding an element to the buffer

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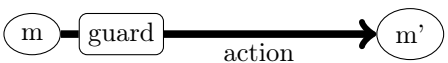
```
def B0.Put : Event (B0 ctx) Unit Unit :=
{
    guard := fun b0 _ =>
         b0.size < ctx.maxSize
    action := fun b0 _ _ =>
         ((),{ size := b0.size + 1 })
}
```

Proof Obligation : *safety*



What is a Proof Obligation ?

- **Users** need to prove them when they specify the system
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safety (m : M) (x : α): Machine.invariant m $\rightarrow$ (grd : ev.guard m x) $\rightarrow$ Machine.invariant (ev.action m x grd).snd

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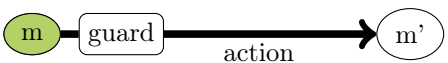

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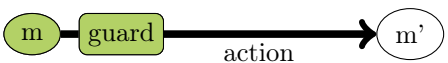

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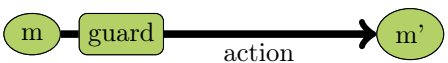

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Proof of the *safety* PO for Put

The steps of the proof

1. **Goal**:
   b0.size ≤ ctx.maxSize → (grd : b0.size < ctx.maxSize)
   → (((), size := b0.size + 1).snd).size ≤ ctx.maxSize

```
safety := fun b0 x =>
by
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   - $hinv : b0.size \leq ctx.maxSize$
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safety := fun b0 x =>
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    intros hinv hgrd

    exact hgrd
```

Refinement between machines

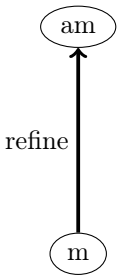
am

m

What is Machine Refinement ?

- Relation between specifications (one is the abstraction of the other)

Refinement between machines



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- refine : AM → M → Prop

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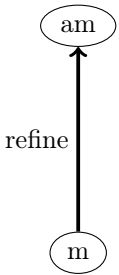

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  → refine am m
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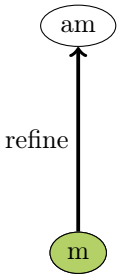

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Example of refinement between machines

Abstract Buffer (B0) : counter

- The context consists of the bound ($ctx.maxSize$) and the property that it is non null ($ctx.maxSizeProp$)
- $size : \mathbb{N}$
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- same context as the abstract one
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The refinement relation

- Correspondance between the two machines : *size* corresponds to the length of *stack*.

```
refine am m    := am.size = m.stack.length

refine_safe am m  :=
    by
      simp[Machine.invariant]
      intros hinv_ccr href
      rw[href]
      exact hinv_ccr
```

Refinement between events

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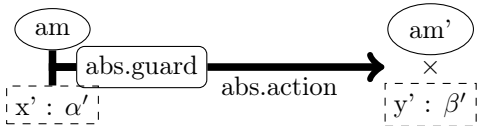
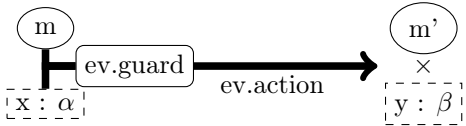
- Each event defined on the concrete machine must refine one of the abstract ones
- To establish such refinement we need :





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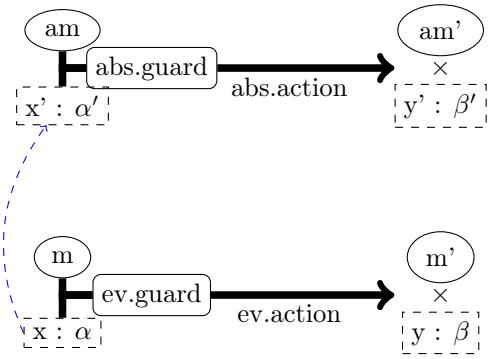

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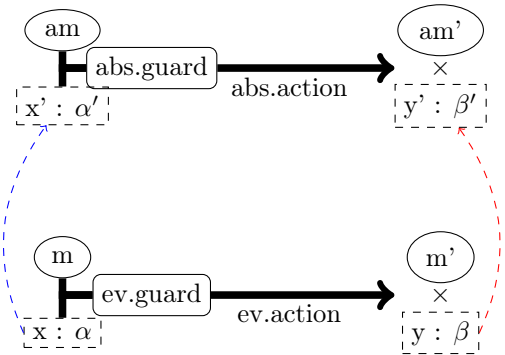

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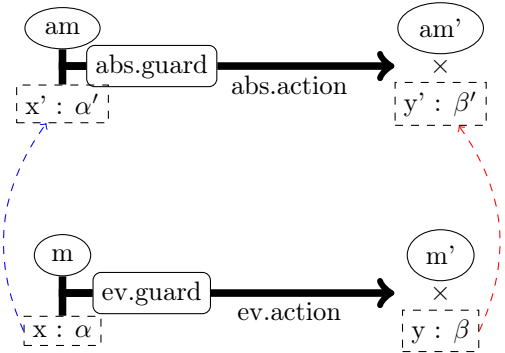

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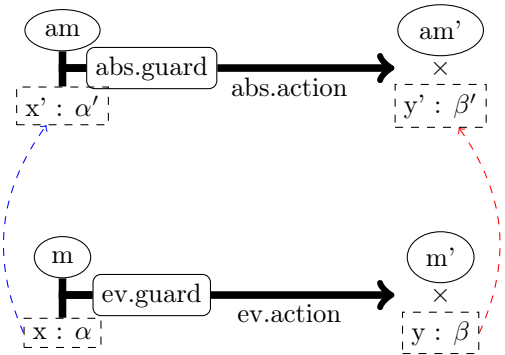
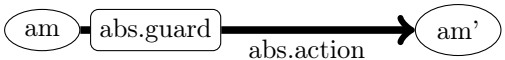
  - A Proof Obligation about the action (*simulation*)



Proof Obligation : *strengthening*



strengthening (m : M) (x : α):
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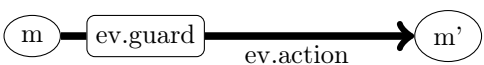
am, refine am m $\rightarrow$ abs.guard am (lift_in x)



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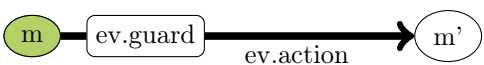

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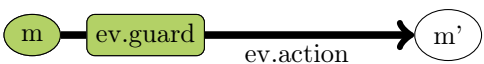

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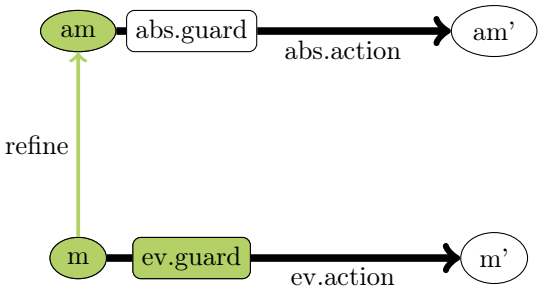

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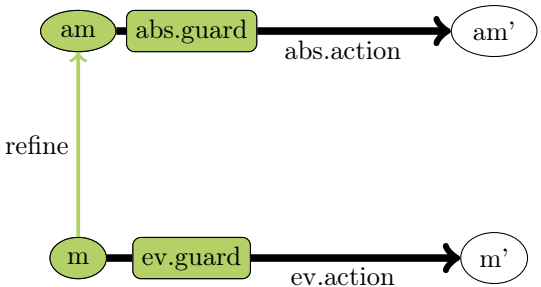

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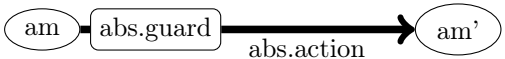
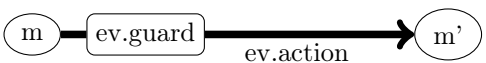
Proof Obligation: *simulation*





simulation (m : M) (x : α):
(Hinv : Machine.invariant m)
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lift_out y = z $\wedge$ refine am' m'

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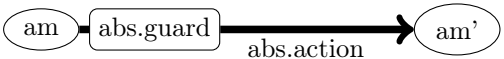

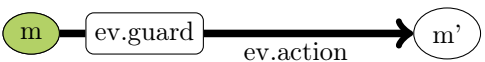

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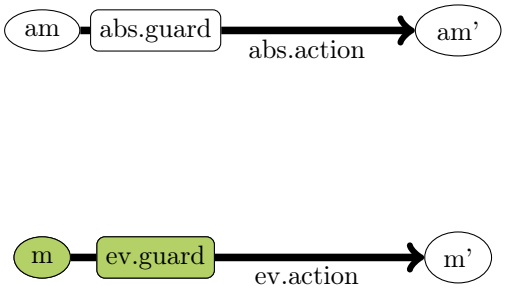

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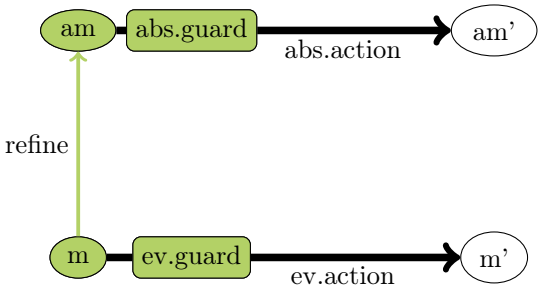

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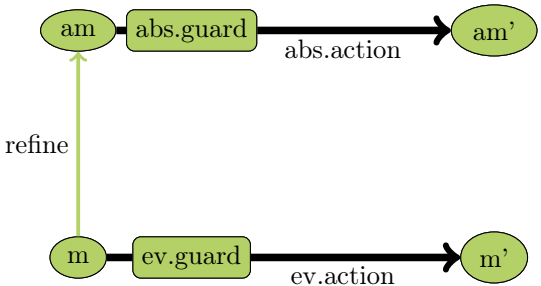

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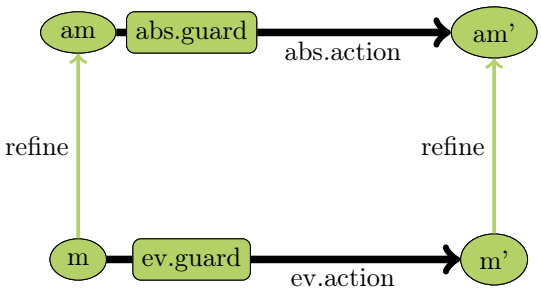

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lift_out y = z $\wedge$ refine am' m'

Example of Event refinement : Put

Add an element to the buffer (B0)

- size := size + 1
- *size* must be stricly below the bound

```
def B0.Put : Event (B0 ctx) Unit Unit :=
{
    guard := fun b0 _ =>
         b0.size < ctx.maxSize
    action := fun b0 _ _ =>
         ((),{ size := b0.size + 1 })
}
```

Add an element to the buffer (B1)

- Insertion at the head of the list
- The length of the list must be strictly below the bound

```
def B1.Put : Event (B0 ctx) α Unit :=
{
    guard := fun b1 _ =>
         b1.length < ctx.maxSize
    action := fun b1 x _ =>
         ((),{ stack := x :: b1.stack })
}
```

Example of Event refinement : Put

Lift of the Inputs/Outputs

- $lift\_in : \alpha \to Unit := \lambda\_.()$
- $lift\_out : Unit \to Unit := \lambda x.x$

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Strengthening

strengthening (m : B1) (x : α):
m.stack.length $\leq$ maxSize
$\to$ m.stack.length $<$ maxSize
$\to \forall$ am : B0, am.size = m.stack.length
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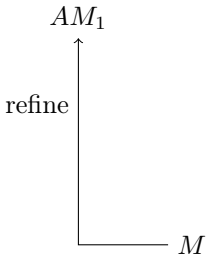
Simulation

simulation (m : B1) (x : α):
(Hinv : m.stack.length $\leq$ maxSize)
$\to$ (Hgrd : m.stack.length $<$ maxSize)
$\to \forall$ am : B0, (Href: am.size = m.stack.length)
$\to$ let (y, m') := ((), stack := x::m.stack)
let (z, am') := ((),size := size +1)

y = z $\wedge$ am'.size = m'.stack.length

What is Multi-refinement

- With *LeanMachines*, we can specify systems through simple refinement (one abstract machine per concrete machine)



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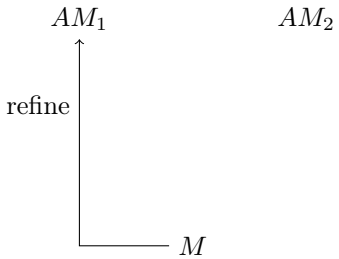
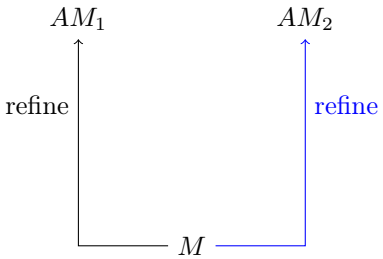


What is Multi-refinement

- With *LeanMachines*, we can specify systems through simple refinement (one abstract machine per concrete machine)
- Sometimes, multiple machines are an abstraction of the same concrete one
- We would like to allow multiple refinement in the framework in order to combine and **reuse** specifications



Multi-refinement : the pushBuffer

[Pes24]

Bounded buffer (B0)

State $size : \mathbb{N}$
Invariant : size $\leq$ SizeMax
Events : Push an element in the buffer

Button

State $pushed : \mathbb{B}$
Invariant : $True$
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Multi-refinement : the pushBuffer

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The pushBuffer

State $stack$: $List\ \alpha$, $pushed : \mathbb{B}$
Invariant : $stack.length \leq max$
Events : push the button (which pushes an arbitrary element of type α to in buffer)

Multiple refinement of Events (example)

Bounded buffer (B0) : Put

guard : $size + 1 \leq SizeMax$

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Button : Push

guard : $\neg pressed$

action : $pressed := true$

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guard : $\neg pressed$
action : $pressed := true$

The pushBuffer : PushAdd

guard : $stack.length + 1 \leq SizeMax \wedge \neg pressed$
action : $stack := x :: stack, pushed := true$

Previous version of the API (refinement)

```
structure _REventPO  [Machine ACTX AM] [Machine CTX M] [instR: Refinement AM M]
   (ev : _Event M α β)  (α' β')
   extends _EventPO ev where
  abstract : _Event AM α' β'
  lift_in : α → α'
  lift_out : β → β'
  strengthening (m : M) (x : α):
  /- [...] -/
  simulation (m : M) (x : α):
  /- [...] -/

structure OrdinaryREvent (AM) [Machine ACTX AM] (M) [Machine CTX M]
[instR: Refinement AM M]
  (α β) (α':=α) (β':=β) extends _Event M α β where
  po : _REventPO (instR:=instR) to_Event
  α' β'
```

The issue with this version

```
def PushBuffer.PushAdd : OrdinaryREvent
    (B0 ctx) (PushBuffer ctx α) α Unit :=
{  guard m _     := /- [...]-/
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- We need the define **everything** about the event (action, guard, safety, refinement PO) at once

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Limits of the use of `structure`

- We need the define **everything** about the event (action, guard, safety, refinement PO) at once
- It's not possible to **combine** structure definitions

⇒ Does not allow *multi-refinement* !

The use of *typeclass* (safe events)

```
class SafeEventPO [Machine CTX M] {α β} (ev : Event M α β)  where
  safety (m : M) (x : α):
    Machine.invariant m
    → (grd : ev.guard m x)
    → Machine.invariant (ev.action m x grd).snd
```

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```

- Proof obligations are thus summed up in a *typeclass* specifying the properties (safety, refinement of events, etc.)
- We add steps to the definition of events :
  1. Define its action and guard
  2. Instanciate the *typeclass* that correspond to what the user wants to prove about the event

The use of *typeclass* (refinement)

```
class SafeREventPO {α β α' β'}
   [Machine ACTX AM] [Machine CTX M]
   [instR: Refinement AM M]
   (ev : Event M α β)
   (abs : Event AM α' β')
   [instSafeAbs : SafeEventPO abs]
   [instSafeEv : SafeEventPO ev]
where
  lift_in : α → α'
  lift_out : β → β'
  strengthening (m : M) (x : α): /- [...]
    -/
  simulation (m : M) (x : α):    /- [...]
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```

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```

- We require that *safety* was discharged through *typeclass* constraints (*instSafeAbs* and *instSafeEv*)
- There can be multiple instanciation of this *typeclass* for the same concrete event !

Coherence with previous API (Events)

```
structure OrdinaryEvent (M)
[Machine CTX M] (α β : Type) where
  guard (m : M) (x : α) : Prop
  action (m : M) (x : α)
   (grd : guard m x) : β × M
  safety (m : M) (x : α) :
    Machine.invariant m
    → (grd : guard m x)
    → Machine.invariant
      (action m x grd).2

instance instSafeEventPO_OrdinaryEvent
    [Machine CTX M]
  (ev : OrdinaryEvent M α β):
    SafeEventPO ev.toEvent
  where
  safety := ev.safety
```

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- We want to keep an API based on structures
- We are able to switch to the use of *typeclass* with *instSafeEventPO_ OrdinaryEvent*

Multiple refinement of Specifications (example)

Bounded buffer (B0)

State $size : \mathbb{N}$
Invariant : size $\leq$ SizeMax
Events : Push an element in the buffer

Button

State $pushed : \mathbb{B}$
Invariant : $True$
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guard : $stack.length + 1 \leq SizeMax \wedge \neg pressed$
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Multiple refinement of Events (example)

```
def PushBuffer.PushAdd : OrdinaryEvent (PushBuffer ctx α) α Unit :=
  {
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    action m x _  := ((),{ stack := x::m.stack, pushed := true})
    safety := by simp[Machine.invariant]
  }
```

Method

Multiple refinement of Events (example)

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1. We define the event PushAdd as an OrdinaryEvent (we use the structure API)

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```

Method

1. We define the event PushAdd as an OrdinaryEvent (we use the structure API)
2. Then we can instanciate the *typeclass SafeREventPO* twice (next slide !)

Multiple refinement of Events (example)

```
instance : SafeREventPO
  PushBuffer.PushAdd.toEvent
  (AM := Button)
  Button.Push.toEvent
  (instSafeEv :=
    instSafeEventPO_OrdinaryEvent
    PushBuffer.PushAdd)
  (instSafeAbs :=
    instSafeEventPO_OrdinaryEvent
    Button.Push)
  where
    lift_in := λ _ => () ; lift_out := id
    simulation    := /- [...] -/
    strengthening := /- [...] -/
```

```
instance : SafeREventPO
  PushBuffer.PushAdd.toEvent
  (AM := B0 ctx)
  B0.Put.toEvent
  (instSafeEv :=
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    PushBuffer.PushAdd)
  (instSafeAbs :=
    instSafeEventPO_OrdinaryEvent B0.Put)
  where
    lift_in := λ _ => () ; lift_out := id
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V. Filou, M. Mosbah, and M. Tounsi, *Towards Proved Distributed Algorithms through Refinement, Composition and Local Computations*,[FMT13]

- "We think that a form of multirefinement would prevent the duplication of proofs and allow better automation of the approach"

Conclusions

Future works

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- Possible extension of this topic towards probabilistic reactive programming (Doctoral Research Project with Christine Tasson, Frédéric Peschanski and Romain Demangeon)

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